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Pricing electricity futures with distortion functions under model ambiguity

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Energieforschungsgespräche Disentis

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Agenda

Motivation

- Model link between price of commodity future $F(t,T)$ with delivery at time T and current spot price $S(t)$ at time t
- Classical approach: Cost of carry concept

$$F(t,T) = S(t)e^{c(T-t)}$$

  where $c = r + u - y$ is the sum of interest rate r and storage cost u minus convenience yield y
- Not applicable for power futures due to non-storability of electricity
- Alternative: Price futures as expected spot price plus risk premium

$$\mathbb{F}(\tau, m) = \mathbb{E}_P(X^m|\mathcal{F}_\tau) + \text{risk premium}$$

  where $\mathbb{F}(\tau, m)$ is the price of the future contract (base/peak) with delivery in month $m = [T_1, T_2]$ and

X^m	average spot price over month m
P	physical measure of X^m
$\mathcal{F}_\tau$	σ-algebra of spot prices until time $\tau < T_1$

Reason for existence of risk premium

- Market participants are risk averse:
  - For consumers, the risk premium is the amount they are willing to pay in addition to the expectation of spot prices to protect themselves against large price increases
  - For producers, it is the minimum discount they are willing to accept as protection against a decrease in spot prices
- Risk preferences change with time-to-delivery (Benth et al. 2008):
  - Consumers are willing to pay a premium for futures with short time-to-delivery
  - Producers accept a discount for the option of fixing the electricity price longer in advance

Literature overview

- Bessembinder/Lemmon (2002): Risk premium negatively related to volatility and positively related to skewness of future spot prices
- Benth et al. (2008) estimate premium within equilibrium framework under the assumption of exponential utility functions for producers and consumers. A forward price is expressed as a risk-neutral expectation after a change the probability measure by an Esscher transform
- Benth/Sgarra (2012) also use an Esscher transform, but point out its limited flexibility
- Benth/Meyer-Brandis (2009) develop a concept to incorporate additional information that is not contained in the σ-algebra $\mathcal{F}_\tau$ of spot prices at trading time τ
- Benth et al. (2013) apply this enlargement of filtration to base futures in the German market
- Veraart/Veraart (2013) and Janczura (2014) apply a change in the pricing measure

Ex-ante vs. ex-post risk premium

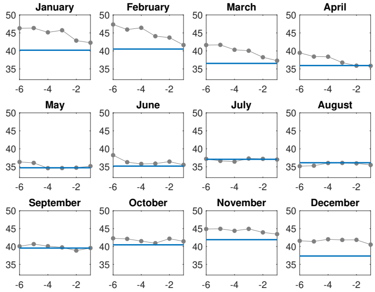
- The risk premium may be studied at trading time τ (ex-ante) or after delivery (ex-post):

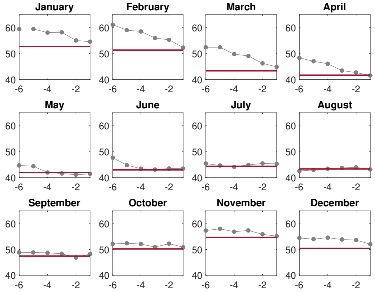
$$r_{ante}(\tau, m) = \mathbb{F}(\tau, m) - \mathbb{E}_P(X^m | \mathcal{F}_\tau)$$
$$r_{post}(\tau, m) = \mathbb{F}(\tau, m) - X_r^m$$

  where X_r^m is the realized monthly average spot price for a certain load profile (base/peak)

- The above mentioned papers model the ex-ante risk premium. A typical finding is that the premium decreases if delivery is further in the future
- Comparison with ex-post premium derived from data shows that premium increases for longer time-to-delivery
- For the analysis in the sequel, we use German (Phelix) base and peak futures traded at the EEX and day-ahead prices for Germany from EPEX

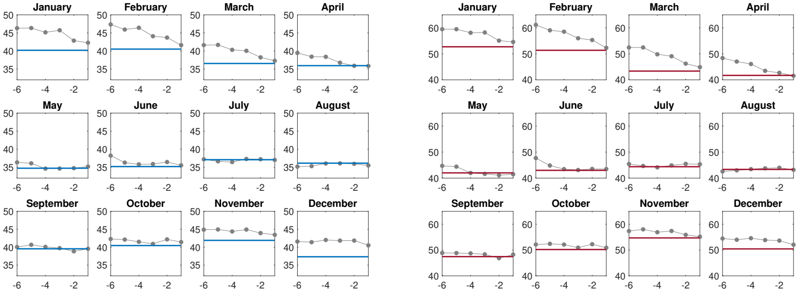
Ex-post risk premium observed in data





- The graph shows for each calendar month between 2010 and 2017 the
  - average evolution of prices $\mathbb{F}(\tau(k,m),m)$ for futures with $k = 6,\dots,1$ months to delivery (dots) in comparison with the
  - averages of realized monthly spot prices (horizontal lines)
- Base futures are shown left, peak futures right
- The differences between the lines correspond to the average ex-post risk premia for different time-to-delivery

Ex-post risk premium observed in data



- Observed risk premium higher for longer time-to-delivery, decreases when delivery is approached
- Differences between seasons, more pronounced for base futures:
  - Larger premia in winter months (October to March)
  - Premia around zero in summer (April to September)

Our approach

- More flexible modelling approach needed:
  - No assumptions on risk preferences, derive only from data
  - Take into account differences between seasons
- Our approach:
  - Risk preferences are modeled by distortion functions (common pricing principle in insurance economics)
  - "Distorting" the physical measure P of X^m (given $\mathcal{F}_\tau$) leads to a new pricing measure P_h, and futures prices can be written as

$$\mathbb{F}(\tau, m) = \mathbb{E}_{P_h}(X^m|\mathcal{F}_\tau)$$

  - We allow for a shift of the distribution of X^m to incorporate additional information not reflected in past spot prices
  - Since any model for future spot prices is imperfect, we also include model risk (ambiguity) in the approach
  - We separate futures with delivery in different seasons

Distortion premium principle

- Idea: Incorporate risk premium by transforming the probability P or cdf F, respectively (Denneberg 1990, Wang 1995)
- The distortion premium is defined as the expectation of a random variable X w.r.t. a distortion of the probability P
- In practice, we distort the cdf of random losses (in our context: spot prices).
- Thus we define the distortion premium applied to the cdf:

$$\pi_h(F) = \int_0^1 F^{-1}(v)h(v)dv$$

  where h is a density on $[0,1]$
- Depending on the distortion, the risk premium can be positive or negative (i.e., $\pi_h(F) \geq \mathbb{E}(F)$ or $\pi_h(F) \leq \mathbb{E}(F)$)

AV@R as example of a distortion premium

- Consider as example the one-step density

$$h(v) = \frac{1}{1-\alpha}\mathbb{1}_{v\geq\alpha}$$

for $0 \leq \alpha < 1$

- This leads to the well-known average value-at-risk (conditional value-at-risk, expected shortfall):

$$\mathrm{AV@R}_\alpha(F) = \frac{1}{1-\alpha}\int_\alpha^1 F^{-1}(v)dv$$

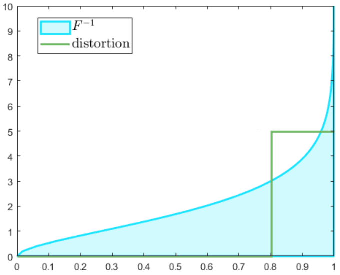


Figure: *Density h for* $AV@R_{0.8}$.

Incorporation of model ambiguity

- Any model that implies a distribution F of future spot prices may be imperfect since the true (but unknown) distribution is G
- We account for potential misspecification by considering a set of models called ambiguity set
- Its radius $\varepsilon > 0$ is calculated with respect to the Wasserstein distance:

$$\pi_h^{\varepsilon}(F) = \sup\{\pi_h(G) \ : \ \mathrm{WD}(F, G) \leq \varepsilon\} = \pi_h(F) \pm \varepsilon \, ||h||_{\infty}$$

- The last term has positive (negative) sign if h is increasing (decreasing)

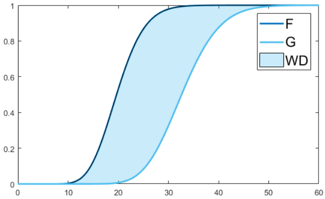


Figure: *Wasserstein distance* $\mathrm{WD} = \int_0^{\infty} |F(x) - G(x)| dx$ *between* F *and* G*.*

Correction of baseline distribution

- Following Benth/Meyer-Brandis (2009), we allow also for a shift in the distribution of X^m
- This may reflect a premium for forward-looking information not reflected in past spot price data
- The correction parameter $\theta > 0$ reduces, $\theta < 0$ increases mean and variance of the baseline distribution

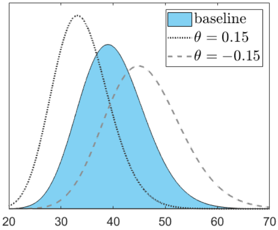


Figure: *Effect of positive and negative shift parameter on baseline distribution.*

Recovering distortion functions under model ambiguity

- Putting all components together, the price of a futures contract with delivery in month m at time τ is:

$$\mathbb{F}(\tau, m) = \underbrace{(1-\theta)}_{\text{correction}} \cdot \underbrace{\pi_h(X_\tau^m | \mathcal{F}_\tau)}_{\text{risk preferences}} \pm \underbrace{\varepsilon \cdot ||h||_\infty}_{\text{ambiguity}}$$

- Identify correction θ, distortion density h and ambiguity radius ε to explain pricing mechanism from the data
- Distortion densities h are approximated by step functions as well as splines
- Futures contracts are grouped by time-to-delivery (one to six months) and season (winter/summer), parameters are estimated individually for each of the 12 groups
- The recovering procedure fits observed futures prices to those implied by the model, subject to conditions ensuring that the estimated $\widehat{h}$ is a valid distortion density
- For the estimation, the baseline distribution is sampled from a spot model

Characteristics of electricity spot prices

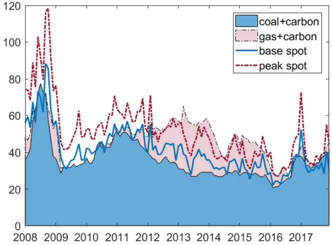


Figure: *Monthly average base and peak spot prices vs. marginal costs of generation with coal and gas (in €/MWh).*

Characteristic features of electricity spot prices:

1. Price levels change over time due to fluctuations in fuel prices
2. Persistent clustering of price spikes (upwards/downwards) over several hours skews the distribution
3. Prices exhibit seasonality patterns (yearly, weekly, intra-day)

Regime switching model for spot prices

- The hourly spot price at time t is described by a Markov regime-switching model with base, lower and upper spike regime:

$$spot_t = \begin{cases} \ell_t^L - \xi_t^L, & \text{if the system is in the lower spike regime,} \\ s_t \cdot \exp(Y_t), & \text{if the system is in the base regime or} \\ \ell_t^U + \xi_t^U, & \text{if the system is in the upper spike regime.} \end{cases}$$

- $\ell_t^U = s_t \cdot L_t \cdot \exp(\alpha_\beta^U)$ and $\ell_t^L = s_t \cdot L_t \cdot \exp(-\alpha_\beta^L)$ are regime limits
- The latent stochastic price level L_t follows a geometric Brownian motion, s_t is a seasonality component
- Y_t is the logarithm of the spot price (corrected for seasonal effects) and follows an Ornstein-Uhlenbeck process around the stochastic mean $\ln L_t$
- Deviations of spot prices from the regime limits in the spike regimes are Weibull-distributed: $\xi_t^U \sim \mathrm{Wei}(\lambda_\gamma^U, k_\gamma^U)$, $\xi_t^L \sim \mathrm{Wei}(\lambda_\gamma^L, k_\gamma^L)$
- Season-dependent transition matrix Π_γ
- $\beta := \beta(t)$ and $\gamma := \gamma(t)$ map time t to an index set in order to use different parameter sets in different seasons, weekdays or time of the day

Estimation results for base futures

(a) Step distortion densities

	label	k	$\widehat{h}$	$\widehat{\theta}$	$\widehat{\varepsilon}$	$\widehat{\varepsilon} \cdot \|\|\widehat{h}\|\|_\infty$
W	-1B	1	$\nearrow$	0.1263	0.1177	1.1770
	-2B	2	$\nearrow$	0.1243	0.1347	1.3472
	-3B	3	$\nearrow$	0.0620	0.2179	1.3634
	-4B	4	$\nearrow$	-0.0083	1.0227	1.4301
	-5B	5	$\nearrow$	-0.0696	1.4758	1.6887
	-6B	6	–	-0.0927	1.9706	1.9706
S	-1B	1	$\nearrow$	0.1488	0.1542	1.0716
	-2B	2	$\nearrow$	0.1802	0.1108	1.1076
	-3B	3	$\nearrow$	0.1301	0.4059	1.1762
	-4B	4	$\nearrow$	0.0446	1.2529	1.3662
	-5B	5	–	0.0091	1.4456	1.4456
	-6B	6	–	-0.0030	1.7652	1.7652

(b) Spline distortion densities

	label	k	$\widehat{h}$	$\widehat{\theta}$	$\widehat{\varepsilon}$	$\widehat{\varepsilon} \cdot \|\|\widehat{h}\|\|_\infty$
W	-1B	1	$\nearrow$	0.1350	0.0600	1.1809
	-2B	2	$\nearrow$	0.1312	0.0761	1.3569
	-3B	3	$\nearrow$	0.0670	0.1169	1.3668
	-4B	4	$\nearrow$	-0.0107	1.0222	1.4287
	-5B	5	$\nearrow$	-0.0637	0.8977	1.6947
	-6B	6	–	-0.0927	1.9706	1.9706
S	-1B	1	$\nearrow$	0.1499	0.1394	1.0724
	-2B	2	$\nearrow$	0.1780	0.0808	1.1067
	-3B	3	$\nearrow$	0.1314	0.3711	1.1788
	-4B	4	$\nearrow$	0.0373	1.2873	1.3591
	-5B	5	–	0.0091	1.4456	1.4456
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- Positive (negative) $\widehat{\theta}$ decreases (increases) mean and variance
- For short (long) time-to-delivery, empirical distribution shifted downwards (upwards) before applying distortion
- Downward shifts more pronounced in summer

Estimation results for base futures

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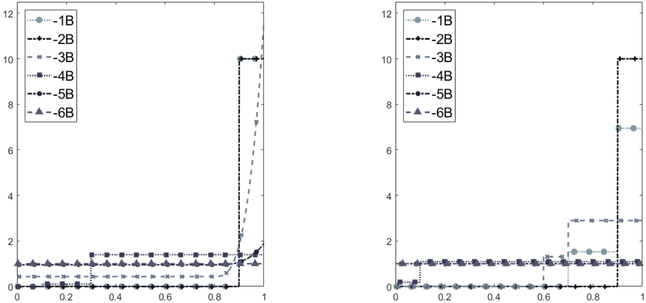
	-3B	3	$\nearrow$	0.1314	0.3711	1.1788
	-4B	4	$\nearrow$	0.0373	1.2873	1.3591
	-5B	5	–	0.0091	1.4456	1.4456
	-6B	6	–	-0.0030	1.7652	1.7652

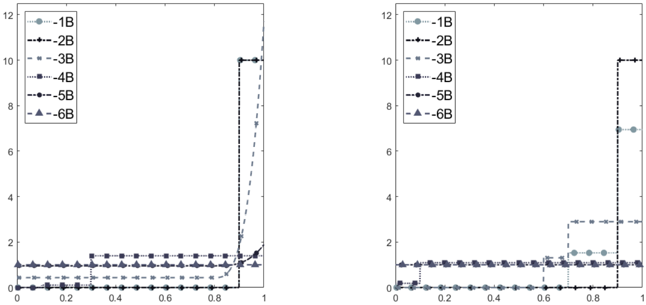
- $\nearrow$ indicates distortion density, implies risk aversion against high prices
- – stands for constant distortion density, no distortion is applied
- Ambiguity increases when delivery is further in the future

Estimated step distortion densities for base futures



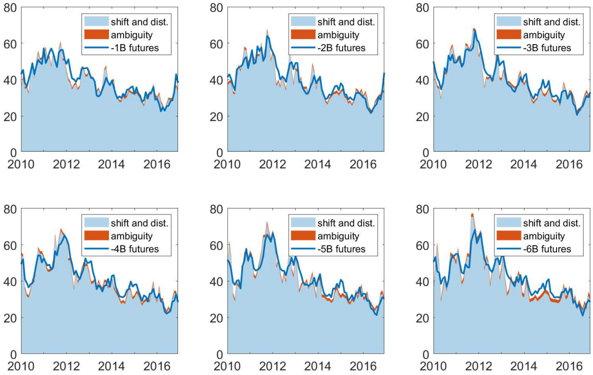
- Increasing distortion densities in winter (left)
- $AV@R_{0.9}$ for delivery in 1 and 2 months reflects risk aversion against high prices for short time-to-delivery
- Higher quantiles become less weighted as time-to-delivery increases, no need for distortion when delivery is 6 months in the future

Estimated step distortion densities for base futures



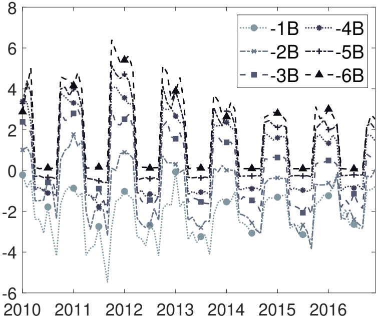
- Similar risk preferences in summer (right), increasing densities for short time-to-delivery ($AV@R_{0.9}$ for delivery in 2 months)
- Higher quantiles become less weighted as time-to-delivery increases
- No need for distortion when delivery is 5 and 6 months in the future

Observed monthly base futures prices vs. estimated prices (€/MWh)



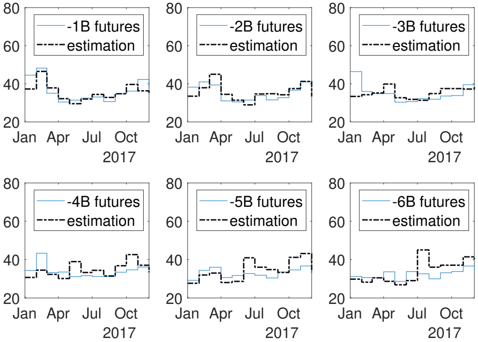
- Estimated futures prices fit observed ones well
- Contribution of ambiguity to overall risk premium small (high explanatory power of spot model for futures prices)

Estimated ex-ante risk premia for base futures (€/MWh)



- Recovered ex-ante risk premia recover persistent seasonal behavior, consistent with observed ex-post premia
- Risk premia for futures with delivery in winter are higher
- Contracts with delivery in summer have negative risk premia (except for shortest time-to-delivery)

Out-of-sample comparison: Oserved monthly base futures prices vs. estimated prices



- Model-implied futures prices match observed ones also out-of-sample
- Seasonal pattern reflected realistically

Summary (1/2)

- We model the link between electricity spot and futures prices, taking into account risk preferences, correction for additional information and ambiguity (model risk)
- The novelty is the application of the distortion pricing principle for futures on non-storable commodities
- Distortion densities weight the probabilities implied by the baseline distribution according to risk preferences
  - Base futures: Increasing densities for short time to delivery
  - Reflects consumers' aversion against large spot prices
  - Peak futures (not shown): No distortion for close deliveries, decreasing density for delivery in summer three months ahead
  - Can be explained by producers' wish to hedge against extreme low prices for large PV infeed
- The shapes of distortion densities are purely identified by the data (no specific assumptions on risk preferences imposed)

Summary (2/2)

- Inclusion of model risk is a further novelty for pricing electricity futures, indicates which spot model should be used
- Spot prices are modeled by a novel regime-switching approach, unobservable factors are estimated by a Kalman filter
- Simulated spot prices have a high explanatory power for futures prices compared to other studies
- Model-implied prices fit observed prices well, the estimated ex-ante premia reflect the pattern of the ex-post premia calculated from observed prices
- Future work:
  - Show contribution of different components by comparing with sub-models
  - Compare pricing approach with benchmark models
  - Illustrate contribution of ambiguity to risk premium by comparison with simpler spot model